Math 55 Discussion problems 16 Feb

1. Show that if $ac \equiv bc \pmod{m}$, where a, b, c, and m are integers with $m > 2$, and $d = \gcd(m, c)$, then $a \equiv b \pmod{\frac{m}{d}}$.

2. Show that an inverse of a modulo m, where a is an integer and $m > 2$ is a positive integer, does not exist if $\gcd(a, m) > 1$.

3. Solve each of these congruences.

   (a) $34x \equiv 77 \pmod{89}$

   (b) $144x \equiv 4 \pmod{233}$

   (c) $200x \equiv 13 \pmod{1001}$

4. Find all solutions to the system of congruences $\begin{cases} x \equiv 2 \pmod{3} \\ x \equiv 1 \pmod{4} \\ x \equiv 3 \pmod{5} \end{cases}$

5. Solve the system of congruence $\begin{cases} x \equiv 3 \pmod{6} \\ x \equiv 4 \pmod{7} \end{cases}$

6. Find all solutions, if any, to the system of congruences $\begin{cases} x \equiv 5 \pmod{6} \\ x \equiv 3 \pmod{10} \\ x \equiv 8 \pmod{15} \end{cases}$

7. Show that the system of congruences $\begin{cases} x \equiv 2 \pmod{6} \\ x \equiv 3 \pmod{9} \end{cases}$ has no solutions.

8. Use Fermat's little theorem to find 23^{1002} **mod** 41.